Unique Rectification Targets in d-Complete Partial Orders

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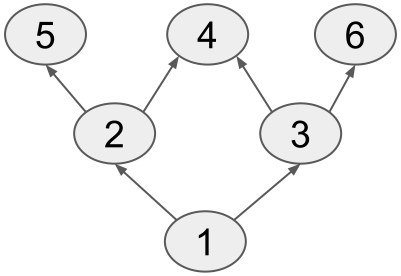
Work supported by the Rutgers Math Department

Motivation

- K Theory
  - associates a number system (ring) with a geometric space
  - Ring illuminates properties of the geometric space
- Geometric Spaces: Flag Varieties
- Our project helps determine multiplication in these number systems

Partially Ordered Sets (Posets)

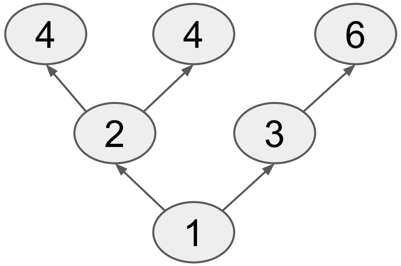
- Assigns hierarchy to elements in sets
- Ancestors of node are “less than” it



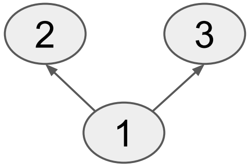
Skewed Poset Graph

- Remove “Order Ideal” from graph
    - Downwardly closed subset of graph

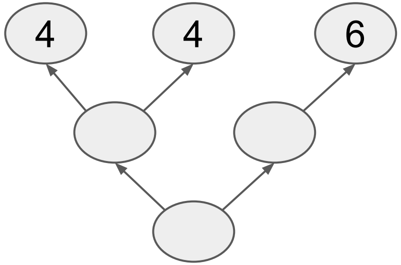
G =



g =

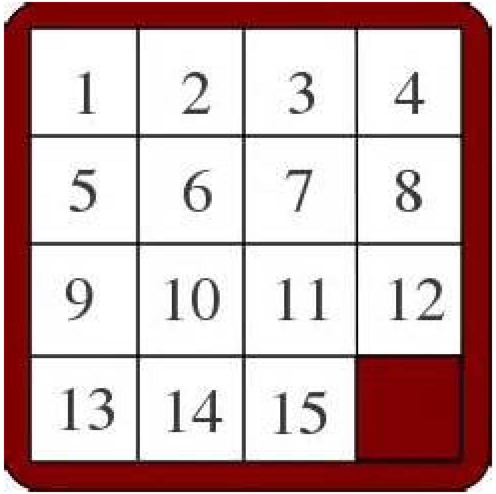


G/g =



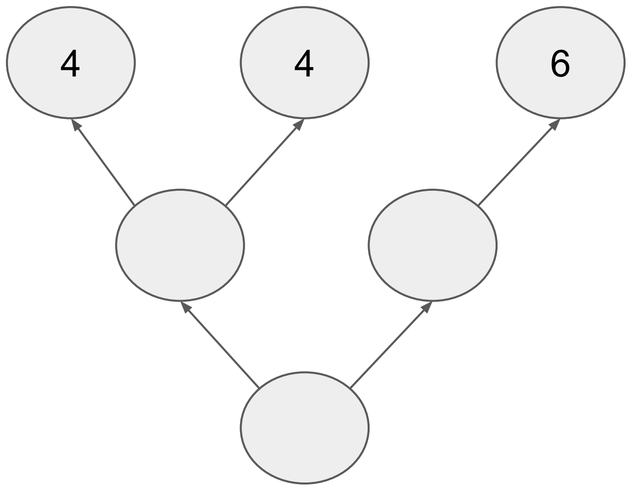
Rectification

- Process to turn a skew shape into a poset of straight shape (an order ideal)
- Algorithm is called Jeu De Taquin - tile game
  - Dot an empty node which is an “inner corner”
    - Inner corner means all of a node's children are filled
  - Swap dot with smallest child (if tie, swap both)
  - Delete dot node when leaf
  - Repeat until no more empty nodes



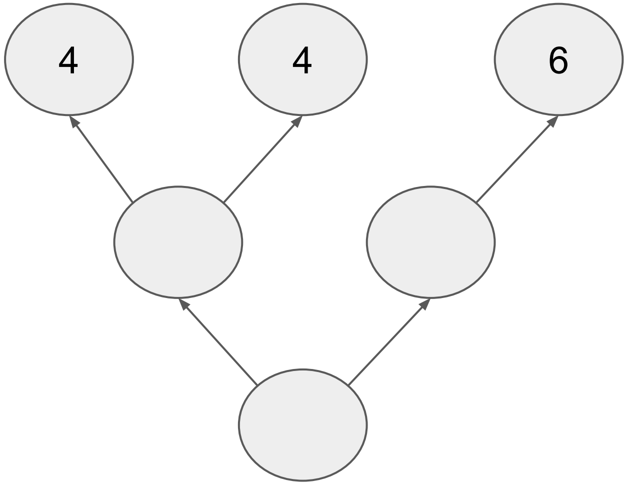
Rectification Example

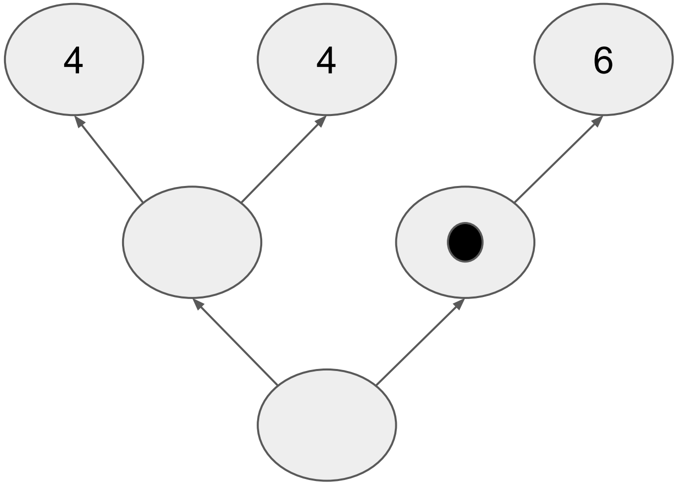
- Begin with skewed graph



Rectification Example

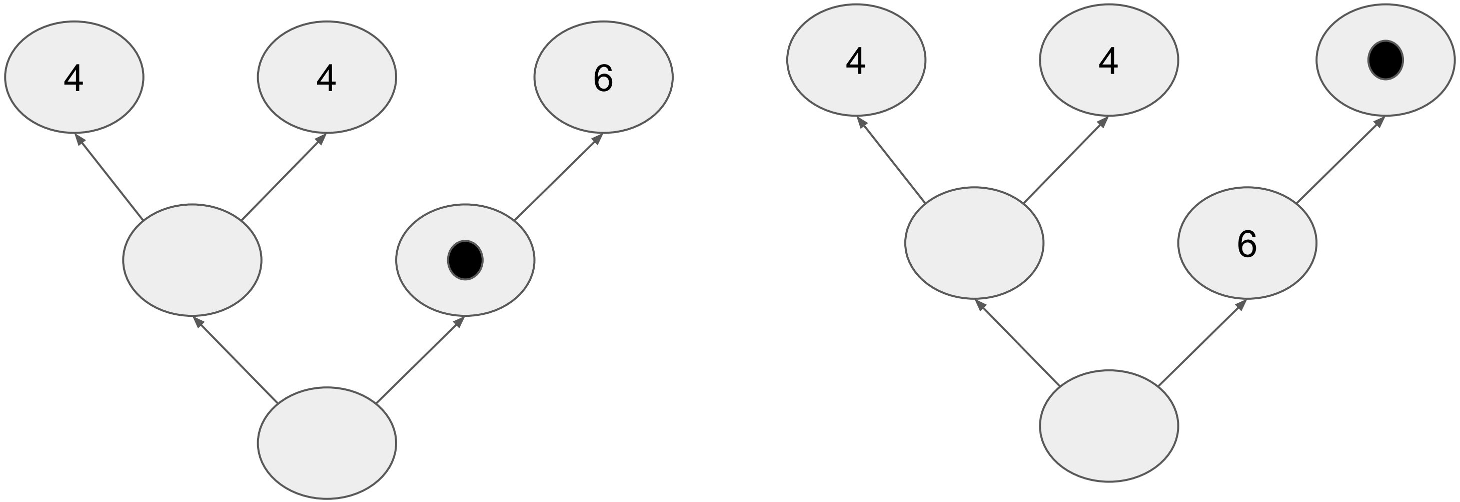
- Fill external empty node with Dot
  - External = Node with children all filled
  - We have choice here





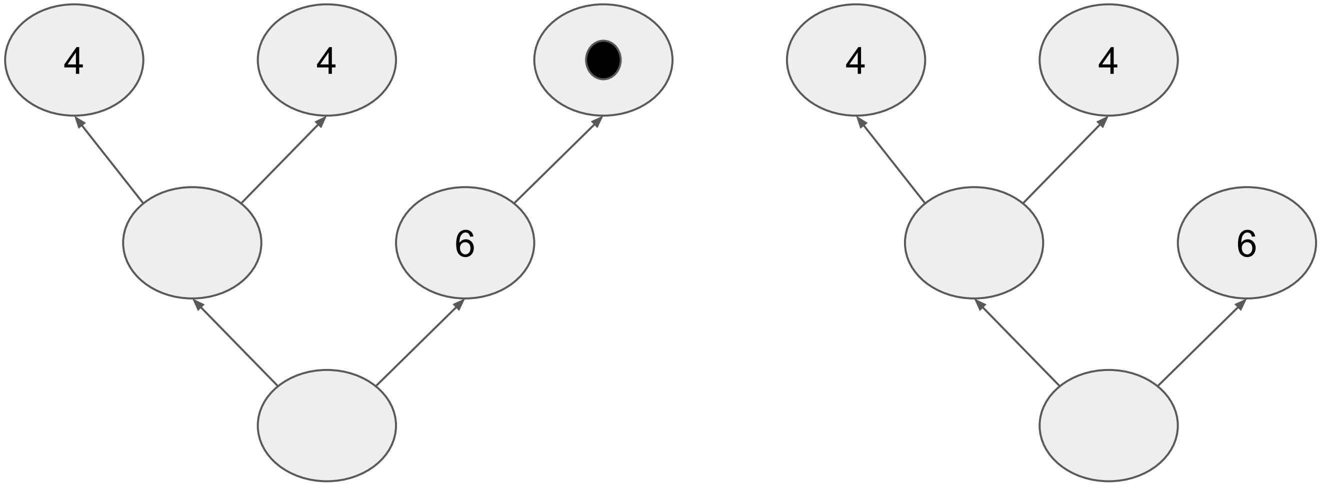
Rectification Example

- Swap dot with smallest child



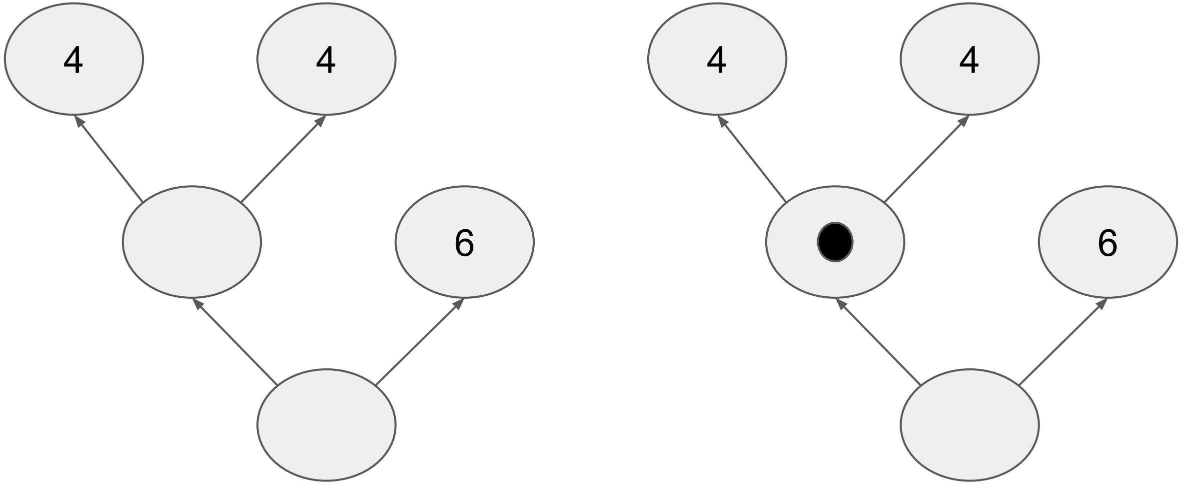
Rectification Example

- Delete dot with no children



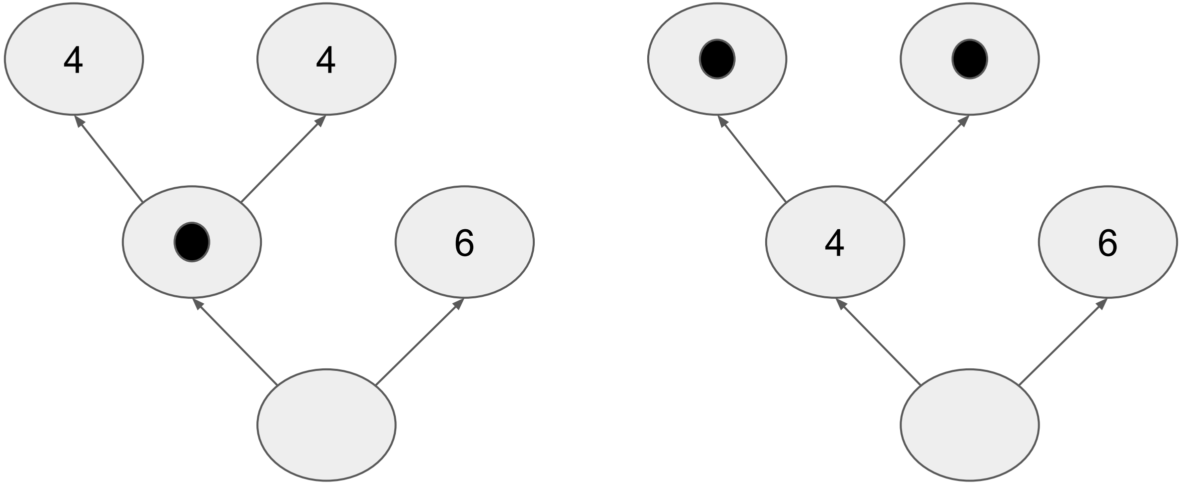
Rectification Example

- Add dot to inner corner



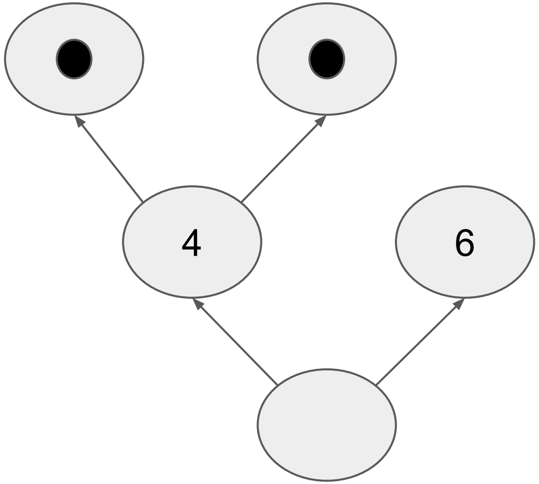
Rectification Example

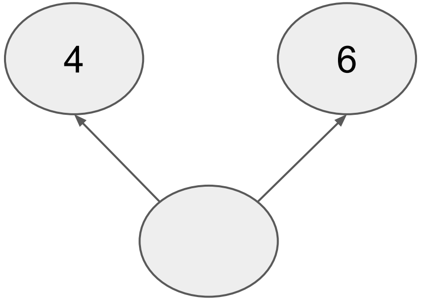
- Swap dot with smallest child (if tie, swap both)



Rectification Example

- Delete dots with no children





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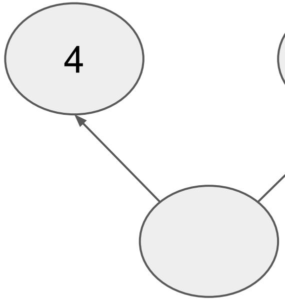
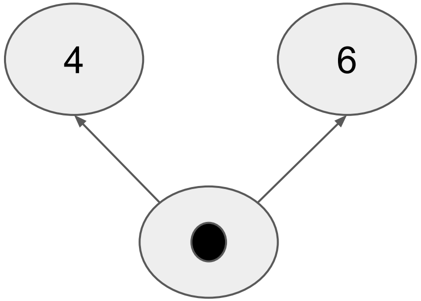


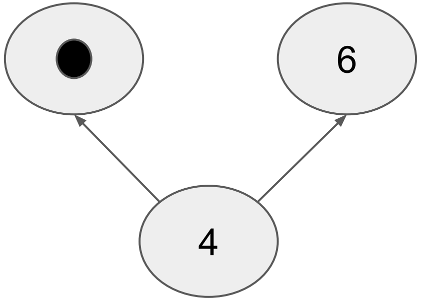


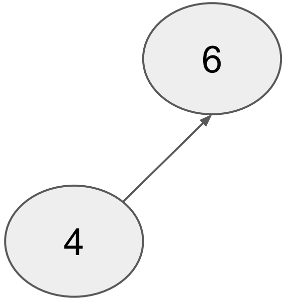
Rectification Example

- Swap dot with smallest child

Rectification Example

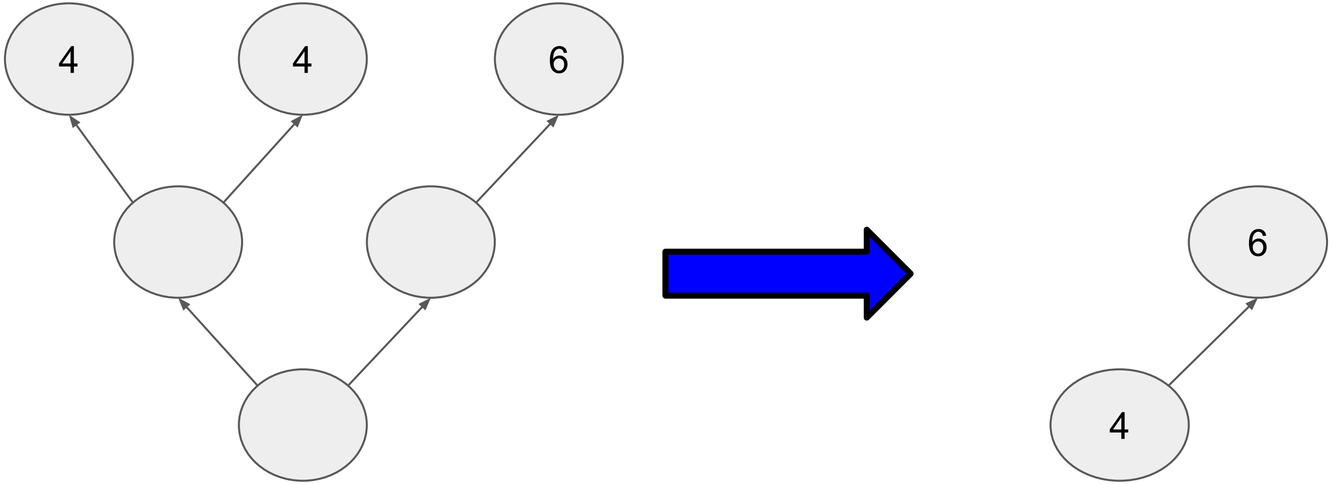
- Delete dot with no children





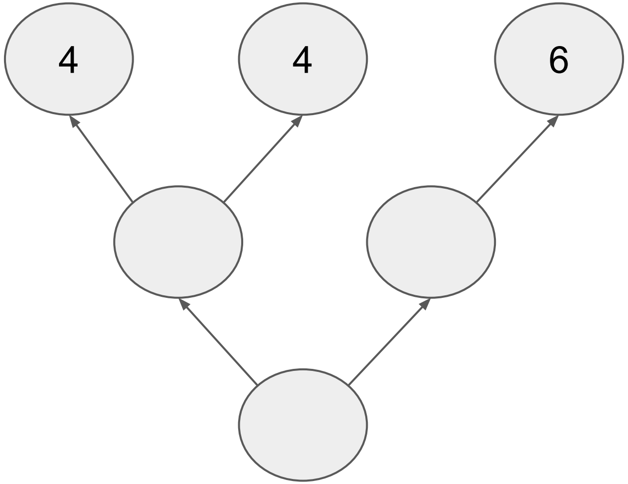
Rectification Example

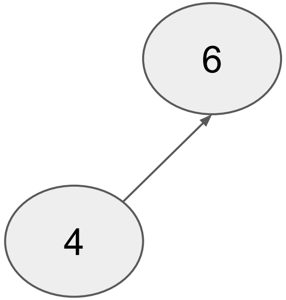
- Done!



Rectification Example

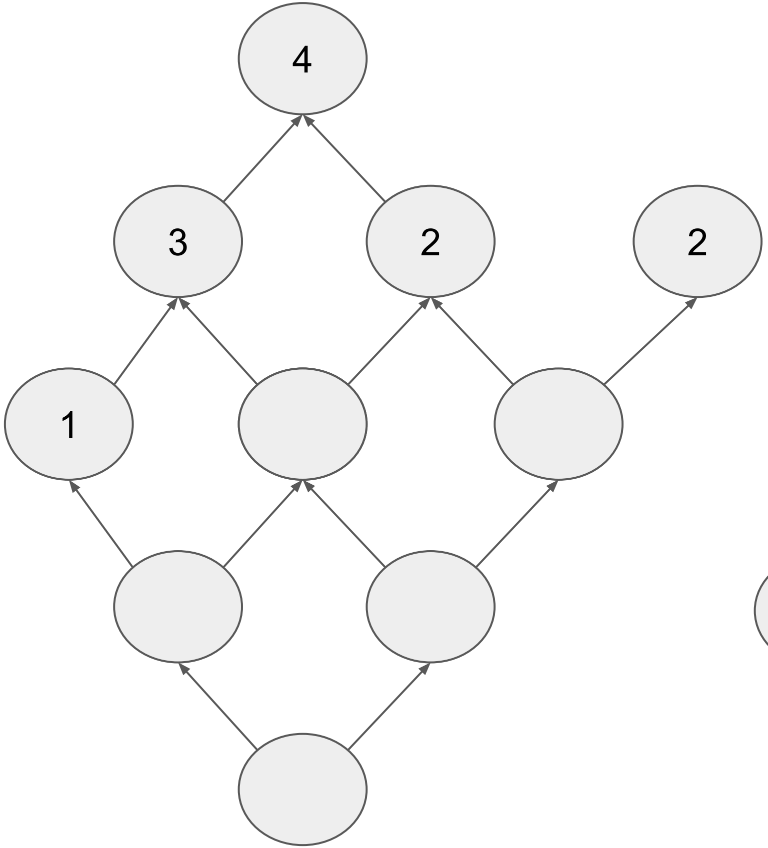
- Did our choices matter?

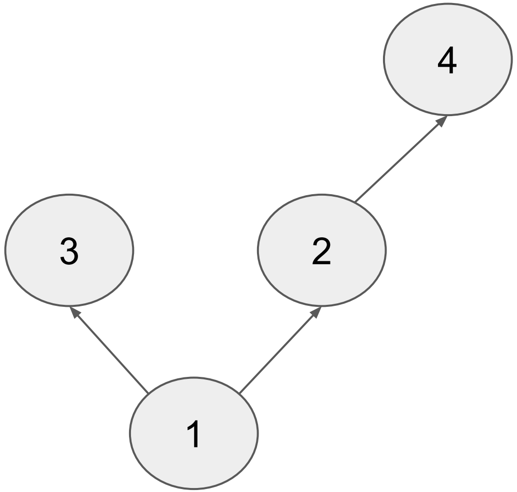


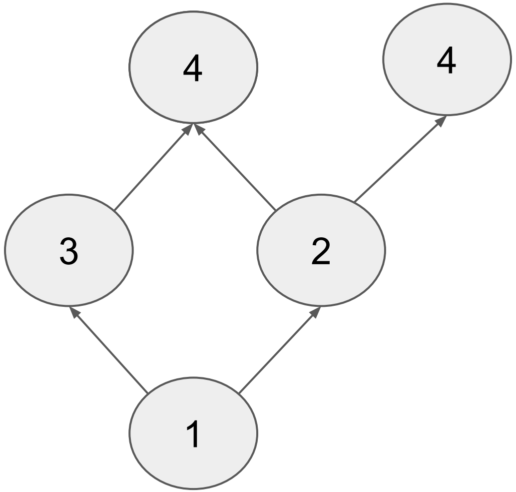


Order Matters?

- Didn't matter that time
- In general can matter







Objective

Definition

A Unique Rectification Target (URT) is a labeling of an order ideal P such that if some labeled skew poset rectifies to P, it will always rectify to P no matter the choices you make during Jeu de Taquin.

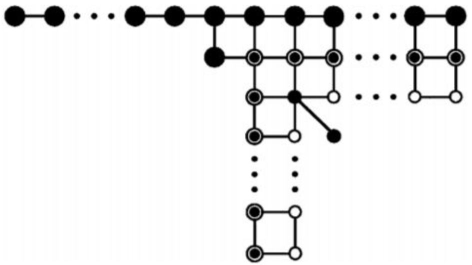
Research Goal

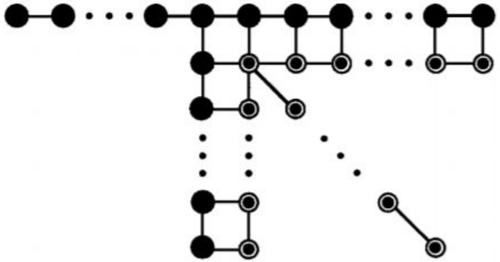
Investigate the existence of URTs on a specific family of posets (d-Complete Posets)

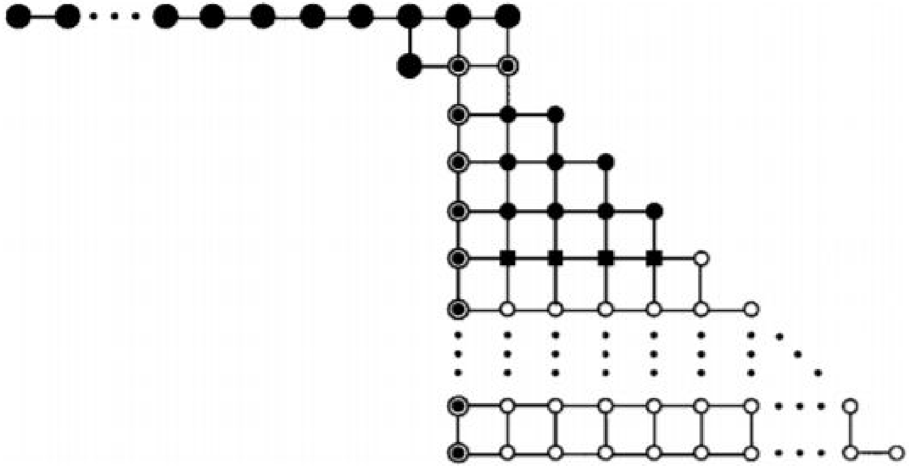
First Result

Let T be a tree. Then every labeling of every ideal of T is a URT.

d-Complete Examples







Why d-complete?

- Lambda-minuscule varieties

Sources

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Thomas H., Yong A (2009). A jeu de taquin theory for increasing tableaux, with applications to K-theoretic Schubert calculus. Algebra Number Theory 3, no. 2, 121--148.

Acknowledgement

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